

RATIO AND PROPORTION

What is ratio?

Ratio is a way of comparing amounts of something. It shows how much bigger one thing is than another. For example:

- Use 1 measure screen wash to 10 measures water
- Use 1 shovel of cement to 3 shovels of sand
- Use 3 parts blue paint to 1 part white

Ratio is the number of **parts** to a mix. The paint mix is 4 parts, with 3 parts blue and 1 part white.

The order in which a ratio is stated is important. For example, the ratio of screenwash to water is 1:10. This means for every 1 measure of screenwash there are 10 measures of water.

Mixing paint in the ratio 3:1 (3 parts blue paint to 1 part white paint) means $3 + 1 = 4$ parts in all.



3 parts blue paint to 1 part white paint = is $\frac{3}{4}$ blue paint to $\frac{1}{4}$ white paint.

If the mix is in the right proportions, we can say that it is in the correct ratio.

Understanding direct proportion

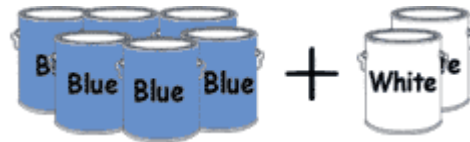
Two quantities are in **direct proportion** when they increase or decrease in the **same ratio**. For example you could increase something by doubling it or decrease it by halving

If we look at the example of mixing paint the ratio is 3 pots blue to 1 pot white, or 3:1.



But this amount of paint will only decorate two walls of a room. What if you wanted to decorate the whole room, four walls? You have to **double** the amount of paint and **increase** it in the **same ratio**.

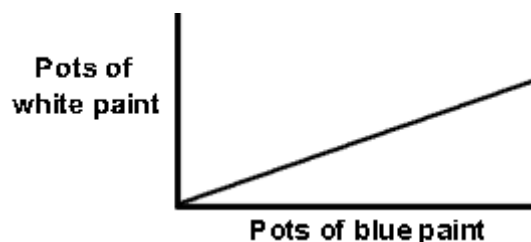
If we **double** the amount of blue paint we need 6 pots.
 If we **double** the amount of white paint we need 2 pots.



The amount of blue and white paint we need increase in direct proportion to each other. Look at the table to see how as you use more blue paint you need more white paint:

Pots of blue paint	3	6	9	12
Pots of white paint	1	2	3	4

Have a look at this graph:



Two quantities which are in **direct proportion** will always produce a graph where all the points can be joined to form a straight line.

Using direct proportion

Understanding proportion can help in making all kinds of calculations. It helps you work out the value or amount of quantities either bigger or smaller than the one about which you have information. Here are some examples:

Example 1:

If you know the cost of 3 packets of batteries is £6.00, can you work out the cost of 5 packets?

To solve this problem we need to know the cost of 1 packet.

If three packets cost £6.00, then you divide £6.00 by 3 to find the price of 1 packet.
 $(6 \div 3 = 2)$

Now you know that they cost £2.00 each, to work out the cost of 5 packets you multiply £2.00 by 5.

$$(2 \times 5 = 10)$$

So, 5 packets of batteries cost **£10.00**

Example 2:

You've invited friends round for a pizza supper. You already have the toppings, so just need to make the pizza base. Looking in the recipe book you notice that the quantities given in the recipe are for 2 people and you need to cook for 5!

Pizza base - to serve 2 people:

100 g flour 60 ml water
4 g yeast 20 ml milk
pinch of salt

The trick here is to divide all the amounts by 2 to give you the quantities for 1 serving. Then multiply the amounts by the number stated in the question, 5.

For **1 serving**, divide by 2:

$$\begin{array}{ll} 100 \text{ g} \div 2 = 50 \text{ g} & 60 \text{ ml} \div 2 = 30 \text{ ml} \\ 4 \text{ g} \div 2 = 2 \text{ g} & 20 \text{ ml} \div 2 = 10 \text{ ml} \end{array}$$

For **5 servings**, multiply by 5:

$$\begin{array}{ll} 50 \text{ g} \times 5 = 250 \text{ g} & 30 \text{ ml} \times 5 = 150 \text{ ml} \\ 2 \text{ g} \times 5 = 10 \text{ g} & 10 \text{ ml} \times 5 = 50 \text{ ml} \end{array}$$

The pinch of salt is up to you!

Simplifying ratios

We can often make the numbers in ratios smaller so that they are easier to compare. You do this by dividing each side of the ratio by the same number, the highest common factor. This is called **simplifying**.

Example:

In a club the ratio of female to male members is 12:18

Both 12 and 18 can be divided by 2.

$$12 \div 2 = 6$$

$$18 \div 2 = 9$$

So a simpler way of saying 12:18 is 6:9.

To make the ratio simpler again, we can divide both 6 and 9 by 3

$$6 \div 3 = 2$$

$$9 \div 3 = 3$$

So a simplest way of saying 12:18 is **2:3**.

These are all **equivalent ratios**, they are in the same proportion. All these ratios mean that for every 2 female members in the club there are 3 males:

12:18
6:9
2:3

2:3 is easier to understand than 12:18!

Tips for ratio and proportion sums

Ratio can be used to solve many different problems, for example recipes, scale drawing and map work.

Changing a ratio

A common test question will ask you to change a ratio - the reverse of cancelling down.

Example:

A map scale is 1 : 25 000. On the map the distance between two shopping centres is 4 cm. What is the actual distance between the shopping centres? Give your answer in km.

A scale of 1 : 25 000 means that everything in real life is 25 000 times bigger than on the map.

So 4 cm on the map is the same as $4 \times 25\,000 = 100\,000$ cm in real life.

Now change the real life distance of 100 000 cm to metres

$$100\,000 \div 100 = 1\,000 \text{ m}$$

And 1 000 m is the same as 1 km.

So the shopping centres are **1 km** apart.

Keeping things in order

When working with ratios **keep both the words and the numbers in the same order** as they are given in the question.

Example:

Share a prize of £20.00 between Dave and Adam in the ratio **3:2**.

The trick with this type of question is to **add** together the numbers in the ratio to find how many parts there are, **divide** by the number of parts to find the value of 1 part, then **multiply** by the number of parts you want to calculate.

- First **add** together the number of parts in the ratio: $3 + 2 = 5$
- **Divide** to find out how much 1 part will be: $£20.00 \div 5 = £4.00$
- To find Dave's share **multiply** $£4.00 \times 3 = £12.00$
- Adam's share is $£4.00 \times 2 = £8.00$
- Dave's £12.00 is $\frac{3}{5}$ of £20.00 (3 of 5 parts).
Adam's £8.00 is $\frac{2}{5}$ of £20.00 (2 of 5 parts).

You can check that you have worked out the ratio correctly by adding the shares together. In this sum Dave's and Adam's shares should equal £20.00

Let's check: $£12.00 + £8.00 = \mathbf{£20.00}$ Correct!

Use the same units

Always check that the things you are comparing are measured in the **same units**.

Example:

Jenna has 75 pence. Hayley has £1.50 What is the ratio of Jenna's money to Hayley's?.

In this problem one amount is in pence, the other in pounds. Before you calculate the ratio you have to make sure they are the **same units**. We have to convert Hayley's amount into pence first.

There are 100 pence to a pound

Hayley's $£1.50 = 150$ pence

So the ratio is $75 : 150$

You can simplify this ratio as both numbers are divisible by 75. The ratio is **1:2**.