Unit 1: NUMBERS AND THEIR USES (I)

This unit will show you how to:

- Add, subtract, multiply and divide integers
- Compare fractions
- Add, subtract, multiply and divide fractions
- Find a fraction of a quantity
- Solve fraction problems
- Use powers and index laws

Keywords	
Number line	Highest (or greatest) common factor (HCF)
Equivalent fractions	Index
Simplest form	Power
Least (or lowest) common multiple (LCM)	Reciprocal
Common denominator	Roots

1.1.- INTEGER NUMBERS

Natural numbers are counting numbers from one to infinity.

We use the letter $\mathbb{N}$ to refer to the set of all natural numbers.

$$\mathbb{N} = \{1, 2, 3, 4, ..., 10, 11, ...\}$$

Whole numbers are counting numbers from zero to infinity.

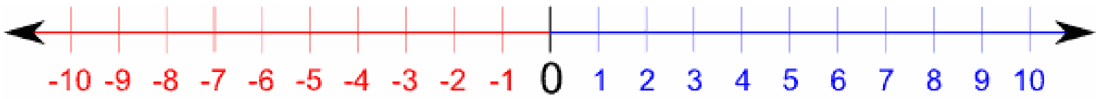
$$\{0, 1, 2, 3, 4, ..., 10, 11, ...\}$$

Integer numbers are positive numbers and negative numbers, but not fractions or decimals.

We use the letter $\mathbb{Z}$ to refer to the set of all integer numbers.

$$\mathbb{Z} = \{..., -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5, ...\}$$

We use a **number line** to show integer numbers:



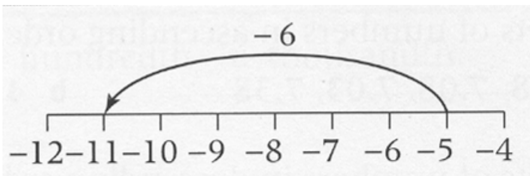
Adding and subtracting integer numbers

- Adding a **positive** number counts as addition. Move right along the number line.
- Subtracting a positive number counts as subtraction. Move left along the number line.
- Adding a **negative** number counts as subtraction. Move left along the number line.
- Subtracting a negative number counts as addition. Move right along the number line.

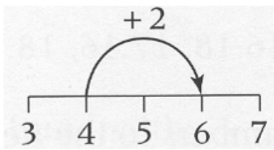
Example:

Work out: a) $(-5)+(-6)$ b) $(+4)-(-2)$ c) $(-7)-(+2)$ d) $(-5)+(+8)$

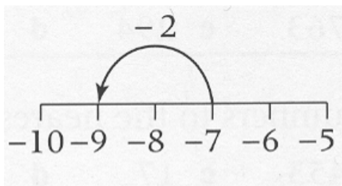
a) Start at -5 on the number line and move 6 places to the left. The answer is -11.



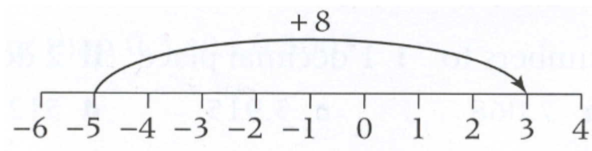
b) Start at +4 on the number line and move 2 places to the right. The answer is +6.



c) Start at -7 on the number line and move 2 places to the left. The answer is -9.



d) Start at -5 on the number line and move 8 places to the right. The answer is +3



In practice:

a) $(-5)+(-6)=-5-6=-11$

b) $(+4)-(-2)=4+2=6$

c) $(-7)-(+2)=-7-2=-9$

d) $(-5)+(+8)=-5+8=3$

Exercise 1:

Find the balance in these bank accounts after the transactions shown:

a) Opening balance £133.45. Deposits of £45.55 and £63.99, followed by withdrawals of £17.50 and £220.
b) Opening balance is -£459.77. Deposit of £6.50, followed by a withdrawal of £17.85.

Exercise 2:

Find the final temperatures in these science experiments:

a) Starting temperature 55°C. It goes up 32°, then down 100°.
b) Starting temperature -15°C. It goes down 28°, increases by 75° and then goes down 17°.
c) Starting temperature -22°C. It goes down 12°, then down 2°, then increases by 53°.

Multiplying and dividing integer numbers

- positive number x positive number = positive number
- positive number x negative number = negative number
- negative number x negative number = positive number

The same rules apply to division.

Example:

$(+4)\cdot(+3)=+12$	$(+50):(+2)=+25$
$(-5)\cdot(+4)=-20$	$(-12):(+6)=-2$
$(-6)\cdot(-3)=+18$	$(-48):(-4)=+12$
$(+7)\cdot(-2)=-14$	$(+24):(-3)=-8$

Properties of Integers

The commutative property of addition:

Changing the **order** of the addends does not change the sum.

$$a+b=b+a$$

Example: $6+4=4+6$

This property does not apply to subtraction or division.

The associative property of addition:

Changing the **grouping** of the addends does not change the sum.

$$(a+b)+c=a+(b+c)$$

$$\text{Example: } (-5+8)+1=-5+(8+1)$$

The distributive property:

Multiplying a sum by a number is the same as multiplying each addend by that number and then adding the two products.

$$a\cdot(b+c)=a\cdot b+a\cdot c$$

$$\text{Example: } -6\cdot(-2+3)=12-18$$

The identity property:

For addition:

Adding 0 and any number does not change the value of the number.

$$a+0=a$$

For multiplication:

Multiplying 1 and any number does not change the value of the number.

$$a\cdot 1=a$$

The inverse property of addition:

The sum of any integer and its additive inverse is 0.

$$a+(-a)=0$$

$$\text{Example: } 9+(-9)=0$$

The inverse property for multiplication does not exist for the set of integers. Fractions are not included in the set of integers.

The zero property of multiplication:

The product of 0 and any number is 0.

$$a\cdot 0=0$$

$$\text{Example: } (-7)\cdot 0=0$$

Order of operations with integers (BEDMAS)

Remember:

1. **B**rackets. ( ) before [ ]
2. **E**xponents (Powers, roots)
3. **D**ivision or **M**ultiplication (left to right)
4. **A**ddition or **S**ubtraction (left to right)

Exercise 3

Compute

a) $-5+4\cdot(-2+1)^3-(-9+6)^2=$

b) $-6-2\cdot\left[-4+5:(-1)\right]=$

c) $12-2\cdot\left[25:(-4-1)+(-2)-(-6-10)\right]=$

d) $-7-(-3)+(-8)\cdot(-1)-(-12):(-4)=$

e) $(-1)^4-(-2)^3+18:(-9)-(-4+2)=$

f) $(-5-4)\cdot(-2)+28:(-7)+(-2)^3=$

g) $-5-4\cdot\left[-8:2-2\cdot(-3)\right]=$

h) $6-5\cdot\left[-4-1+(-2)^2-3^2\right]=$

i) $12-8\cdot\left[-2+4:(-1)-(-3+2)^4\right]=$

j) $(-2)^5:(3+1)^2+2\cdot(-5-4+3)=$

k) $10-10\cdot\left[-6+5\cdot(-4+7-3)\right]=$

1.2.- FRACTIONS

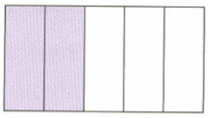
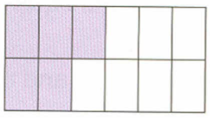
A **fraction** is a number that indicates a part of a unit or a part of a quantity.

Fractions are written in the form $\frac{a}{b}$ where **a** and **b** are whole numbers, and the number b is not 0.

The **denominator** (b) shows how many equal parts the whole has been split into.

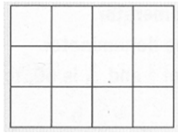
The **numerator** (a) tells us how many of those equal parts are being described.

Examples:

 $\frac{2}{5}$  $\frac{5}{12}$

Exercise 4

Sketch seven copies of the diagram shown.



Shade your diagrams to represent each of these fractions.

a) $\frac{1}{2}$ b) $\frac{2}{3}$ c) $\frac{3}{4}$ d) $\frac{5}{6}$ e) $\frac{7}{12}$ f) $\frac{17}{12}$

<u>Remember</u>:

Proper fraction: numerator is less than the denominator.

Example: $\frac{4}{5}$

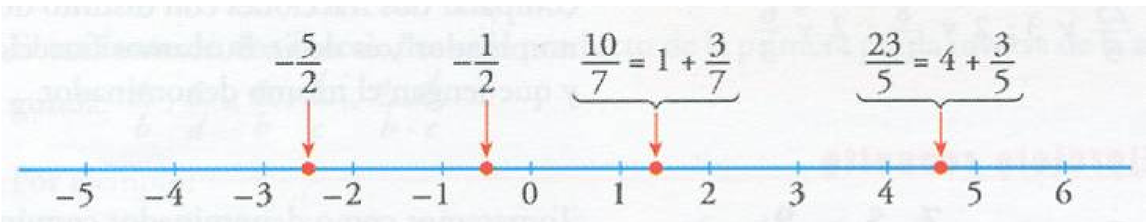
Improper fraction: numerator is greater than or equal to denominator.

Example: $\frac{13}{5}$

Mixed number: whole number and a fraction.

Example: $1\frac{5}{12} = 1 + \frac{5}{12}$

We can also use the number line to show fractions:



Exercise 5

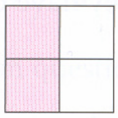
Represent the following fractions in the number line.

$\frac{17}{3}, \ -\frac{11}{4}, \ \frac{20}{5}, \ \frac{2}{3}, \ \frac{16}{7}, \ -\frac{21}{5}, \ -\frac{7}{2}$

Equivalent fractions

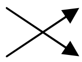
Equivalent fractions are different fractions which express the same amount.

Examples: $\frac{1}{2}$ is equivalent to $\frac{2}{4}$, because $\frac{1}{2}=\frac{2}{4}=0.5$

The diagram shows that $\frac{1}{2}=\frac{2}{4}$ 

$\frac{12}{20}$ is equivalent to $\frac{3}{5}$, because $\frac{12}{20}=\frac{3}{5}=0.6$

We can test if two fractions are equivalent by cross-multiplying their numerators and denominators. This is also called taking the **cross-product**.

$\frac{a}{b}$  $\frac{c}{d}$

For example, $\frac{12}{20}$ and $\frac{24}{40}$ are equivalent because $12\cdot 40=20\cdot 24=480$

We can find equivalent fractions by multiplying or dividing the numerator and the denominator by the same number.

We can **simplify** a fraction by dividing the numerator and the denominator by a common factor. This process is called **"cancelling down"**.

When the fraction cannot be simplified, we say it is written in its **simplest form**.

We can also multiply both numerator and denominator by the same value. Then we are **"amplifying"** fractions.

Exercise 6

Cancel each of these fractions down to their simplest terms.

a) $\frac{2}{4}$ b) $\frac{15}{20}$ c) $\frac{8}{10}$ d) $\frac{95}{100}$ e) $\frac{6}{8}$

Exercise 7

Rewrite each fraction with the denominator shown.

a) $\frac{2}{3}=\frac{\quad}{30}$ b) $\frac{3}{7}=\frac{\quad}{42}$ c) $\frac{7}{9}=\frac{\quad}{45}$ d) $\frac{5}{8}=\frac{\quad}{40}$

Comparing fractions

To compare fractions, two or more fractions:

- Find the **common denominator**.
- Work out the equivalent fractions.
- Write the fractions in ascending or descending order.

Example: Which fraction is bigger $\frac{2}{5}$ or $\frac{5}{12}$?

The common denominator of $\frac{2}{5}$ and $\frac{5}{12}$ is 60, the **LCM (least common multiple)** of 5 and 12.

$\frac{2}{5}=\frac{24}{60}$ (Multiply numerator and denominator by 12)

$\frac{5}{12}=\frac{25}{60}$ (Multiply numerator and denominator by 5)

The order is $\frac{2}{5}<\frac{5}{12}$, so $\frac{5}{12}$ is bigger than $\frac{2}{5}$.

Exercise 8

Write each set of fractions in ascending order. Show your working.

a) $\frac{2}{3}, \frac{1}{5}$ and $\frac{2}{15}$ b) $\frac{1}{4}, \frac{5}{5}$ and $\frac{7}{20}$ c) $\frac{3}{7}, \frac{3}{8}$ and $\frac{5}{14}$ d) $\frac{2}{3}, \frac{5}{6}$ and $\frac{2}{7}$

Exercise 9

Write each set of fractions in descending order. Show your working.

a) $\frac{2}{5}, \frac{1}{2}, \frac{3}{10}$ and $\frac{1}{4}$ b) $\frac{1}{4}, \frac{3}{20}, \frac{4}{5}$ and $\frac{1}{10}$

c) $\frac{2}{5}, \frac{3}{8}, \frac{3}{4}$ and $\frac{17}{40}$ d) $\frac{5}{6}, \frac{11}{24}, \frac{7}{12}$ and $\frac{5}{8}$

Adding and subtracting fractions

You can only add and subtract fractions if they have common denominators.

$$\frac{2}{8}+\frac{5}{8}=\frac{7}{8}$$

If the fractions have different denominators, change them to equivalent fractions with the same denominator, then add.

$$\frac{11}{12}-\frac{1}{3}=\frac{11}{12}-\frac{4}{12}=\frac{7}{12}$$

If your answer is an improper fraction, change it to a mixed number.

$$\frac{3}{4}+\frac{2}{5}=\frac{15}{20}+\frac{8}{20}=\frac{23}{20}=1+\frac{3}{20}=1\frac{3}{20}$$

Cancel any common factors in the numerator and denominator.

$$\frac{4}{5}-\frac{3}{10}=\frac{8}{10}-\frac{3}{10}=\frac{5}{10}=\frac{1}{2}$$

Exercise 10

Do the following calculations and express the answer in its simplest form. Then, change the result into a mixed number.

a) $3-\frac{7}{6}-\frac{3}{4}=$ b) $1\frac{3}{4}+3\frac{1}{3}-2\frac{5}{6}=$

Multiplying and dividing fractions

To multiply fractions, multiply the numerators and then the denominators, then cancel any common factors.

$$\frac{4}{9}\cdot\frac{3}{5}=\frac{12}{45}=\frac{4}{15}$$

The **multiplicative inverse** of an integer is it **reciprocal**. For example, the reciprocal of 5 is $\frac{1}{5}$.

When you multiply a number by its reciprocal you always get 1: $5\cdot\frac{1}{5}=1$

The multiplicative inverse or the reciprocal of a fraction is the original fraction **'turned upside down'**. The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$. $\left(\frac{3}{5}\cdot\frac{5}{3}=1\right)$

To divide two fractions, multiply the first by the reciprocal of the second.

$$\frac{7}{4}:\frac{5}{6}=\frac{7}{4}\cdot\frac{6}{5}=\frac{42}{20}=\frac{21}{20}=1\frac{1}{20}$$

(This is the same as cross-multiplying their numerators and denominators).

- Multiplying by a **unit fraction** (a fraction that has a numerator of 1) is the same as dividing by its denominator. For example, multiplying by $\frac{1}{5}$ is the same as dividing by 5.

$$10 \cdot \frac{1}{5} = \frac{10}{5} = 2 \quad \text{and} \quad 10 : 5 = 2$$

- Dividing by a unit fraction is the same as multiplying by its denominator.

$$10 : \frac{1}{2} = 10 \cdot 2 = 20$$

Exercise 11

Calculate, giving your answers in their simplest form.

a) $\left(\frac{3}{4}+\frac{7}{6}-\frac{7}{8}\right):\frac{25}{12}$ b) $\left(\frac{13}{15}-\frac{7}{25}\right)\cdot\left(\frac{9}{22}+\frac{-13}{33}\right)$ c) $\dfrac{\frac{1}{2}-\left(\frac{3}{4}-1\right)}{\frac{3}{4}+1}$

d) $\dfrac{(-3)\left(\frac{3}{5}-\frac{1}{3}\right)}{(-2)\left(\frac{4}{3}-\frac{6}{5}\right)}$ e) $\dfrac{3-\frac{1}{4}\cdot\left(\frac{3}{5}-\frac{2}{15}\right)}{6+\frac{4}{25}\cdot\left(\frac{1}{2}-\frac{3}{4}\right)}$ f) $\dfrac{\left(\frac{2}{3}-\frac{5}{9}\right)\cdot\left(\frac{3}{4}-\frac{5}{6}\right)}{\left(\frac{7}{12}-\frac{5}{6}\right)\cdot\frac{4}{3}+1}$

Finding fractions of quantities

a) Find $\frac{1}{5}$ of £30 b) Find $\frac{4}{5}$ of £30

To find **one fifth** of a number we **divide the number by five**.

$$\frac{1}{5} \text{ of } £30 = 30 : 5 = £6$$

Then, to find **four fifths** of a number, we first find one fifth of that number and then **multiply this by four**.

$$\frac{1}{5} \text{ of } £30 = 30 : 5 = £6\ ; \qquad £6 \cdot 4 = £24$$

Dividing 30 by 5 and multiplying the number you get by 4 is the same as multiplying 30 by $\frac{4}{5}$.

Therefore, you find fractions of a quantity by multiplying.

For example, two thirds of 5 $= \frac{2}{3} \cdot 5 = \frac{2 \cdot 5}{3} = \frac{10}{3}$

You can extend this method **to finding a fraction of a fraction of a quantity**.

Example:

Tom has £42. He spends $\frac{1}{3}$ of it on Monday. On Tuesday he spends $\frac{3}{4}$ of the remainder. How much does he spend on Tuesday?

Tom spends $\frac{1}{3} \cdot$£42 on Monday, so he has $\frac{2}{3} \cdot$£42 on Tuesday.

$$\frac{3}{4} \cdot \frac{2}{3} \cdot 42 = £21$$

Now, we move on the "inverse problem":

The two thirds of a quantity are 400. What is the quantity?

If the two thirds of a quantity are 400, one third of this quantity is $400 : 2 = 200$.

$\frac{1}{3}$ of a quantity is 200 $\Rightarrow$ the whole quantity is $200 \cdot 3 = 600$

Dividing 400 by 2 and multiplying the number you get by 3 is the same as **multiplying 400 by $\frac{3}{2}$** (the multiplicative inverse of $\frac{2}{3}$).

To sum up:

The fraction $\frac{a}{b}$ of C is equal to $\frac{a}{b} \cdot C$

If the fraction $\frac{a}{b}$ of C is equal to P, then C is equal to $P \cdot \frac{b}{a}$

Exercise 12

In a group of 160 students, $\frac{5}{8}$ were female. How many students were female?

Exercise 13

15 dogs out of a group of 60 have short tails. What fraction of the dogs are short tails?

Exercise 14

Out of a group of 40 people in a shop, 32 were aged thirty or over. What fraction of the people were under thirty?

Exercise 15

This list gives the numbers of trees in a small wood:

beech: 32, oak: 74, elm: 2, ash: 15, chestnut: 15, yew: 1.

List each type as a fraction of the number of trees in the wood.

Exercise 16

A reel holds 60 m of wire when new. $\frac{2}{5}$ of the wire has been used.

a) What length of wire has been used?
b) What length of wire is left on the reel?

Exercise 17

An empty swimming pool is to be filled with water. It takes 12 hours to fill the pool, and the full pool contains 98 m^3 of water. How much water will the pool contain after 5 hours? Show your working.

Exercise 18

Of the people invited to the party, $\frac{1}{4}$ could not come because of illness and $\frac{2}{5}$ could not come because of transport problems. What fraction of those invited could not come?

Exercise 19

$\frac{3}{4}$ of refugees were female. $\frac{2}{5}$ of refugees were girls of 19 years or younger. What fraction of the refugees were women over 19?

Exercise 20

$\frac{4}{5}$ of the cars on the road are saloons. Of these saloons $\frac{1}{8}$ are red. What fraction of the cars on the road are red saloons?

Exercise 21

Sadie has already driven $\frac{13}{28}$ of the distance between college and home. She wants to split the remaining distance into 5 equal parts. What fraction of the whole journey is each part?

Exercise 22

John eats $\frac{2}{5}$ of a bar of chocolate. Linda eats $\frac{4}{9}$ of what remains. What fraction of the bar of chocolate have they eaten between them?

Exercise 23

Use a mental or written method to work out these.

a) The two thirds of a number are 22. What is the number?
b) The five quarters of a number are 35. What is the number?
c) The seven tenths of a quantity are 210. What is the quantity?

Exercise 24

Out of a deposit of oil you empty one half. Out of what remains, you empty one half again, and then you empty $\frac{11}{15}$ of what remains. Finally, there are 36 litres left. How many litres were there at the beginning?

Exercise 25

Complete this magic square. All the rows, columns and diagonals must add up to the same number.

		3/8
1/2	3/4	1

1.3.- POWERS

Repeated multiplications such as 2 · 2 · 2 · 2 can be written in **index** notation as 2^4.

You read 2^4 as **'two to the power of 4'**.

You use powers when factorising, for example: $24 = 2 \cdot 2 \cdot 2 \cdot 3 = 2^3 \cdot 3$

In a power, the number or expression used as a factor is called **base**.

And the number that indicates how many times the base is used as factor is called **exponent** (or **index**).

$$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$$

Base: 2 Exponent: 4

Properties of powers

Powers of the same number can be multiplied and divided.

When multiplying, you add the indices.

$$5^3 \cdot 5^4 = (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5 \cdot 5) = 5^7$$
$$5^{3+4} = 5^7$$

When dividing, you subtract the indices.

$$\frac{3^6}{3^2} = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3} = 3 \cdot 3 \cdot 3 \cdot 3 = 3^4$$
$$3^{6-2} = 3^4$$

And when finding a 'power of a power', you multiply the indices.

$$\left(5^2\right)^3 = 5^2 \cdot 5^2 \cdot 5^2 = 5^{2+2+2} = 5^6$$
$$5^{2 \cdot 3} = 5^6$$

- Simplified, the **index laws** are:

$$a^m \cdot a^n = a^{m+n} \qquad a^m : a^n = a^{m-n} \qquad \left(a^m\right)^n = a^{m \cdot n}$$

(in all the properties, letters are used to represent numbers)

- $(a \cdot b)^n = a^n \cdot b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- Any number (except 0) raised to the power of zero is equal to 1. $a^0 = 1$ *Proof:* $a^0 = a^{n-n} = \frac{a^n}{a^n} = 1$

Examples: $7^0 = 1$ $10^0 = 1$

(0^0 is not defined, it doesn't actually means anything)

- Any number to the power of 1 is just the number itself. $a^1 = a$
- In general, $a^{-n} = \frac{1}{a^n}$.

Proof: $a^n \cdot a^{-n} = a^0 = 1 \Rightarrow a^{-n} = \frac{1}{a^n}$

Remember that the **reciprocal** of a number is 1 divided by that number.

Ex.: Reciprocal of $10 = \frac{1}{10} = 0.1$ Reciprocal of $4^2 = \frac{1}{4^2} = \frac{1}{16} = 0.0625$

So, a **negative index** represents the reciprocal of a number.

Examples: $8^{-1} = \frac{1}{8} = 0.125$ $10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$

- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Proof: $\left(\frac{a}{b}\right)^{-n} = \frac{1}{\left(\frac{a}{b}\right)^n} = \frac{1}{\frac{a^n}{b^n}} = \frac{b^n}{a^n} = \left(\frac{b}{a}\right)^n$

Example: $\left(\frac{5}{3}\right)^{-4} = \left(\frac{3}{5}\right)^4$

Exercise 26

Compute.

a) $(-3)^3$ b) $(-2)^4$ c) $(-2)^{-3}$ d) -3^2 e) -4^{-1}

f) $(-1)^{-2}$ g) $\left(\frac{1}{2}\right)^{-3}$ h) $\left(-\frac{1}{2}\right)^{-2}$ i) $\left(\frac{4}{3}\right)^0$ j) $\left(\frac{3}{5}\right)^{-1}$

Exercise 27

Work out these, giving your answers in index form.

a) $\left(\frac{3}{4}\right)^{-3} : \left(\frac{3}{4}\right)^2$ b) $\frac{2^5 \cdot 2^{-7}}{2^{-4}}$ c) $\left[\left(\frac{1}{2}+1\right)^{-1}\right]^3$ d) $\left(\frac{1}{2}\right)^3 : \left(\frac{1}{4}\right)^2$

Exercise 28

Calculate using the properties of powers.

a) $\dfrac{6^4 \cdot 8^2}{3^2 \cdot 2^3 \cdot 2^4}$ b) $\dfrac{15^2 \cdot 4^2}{12^2 \cdot 10}$ c) $\dfrac{2^{-5} \cdot 4^3}{16}$

d) $\dfrac{2^5 \cdot 3^2 \cdot 4^{-1}}{2^3 \cdot 9^{-1}}$ e) $\dfrac{6^2 \cdot 9^2}{2^3 \cdot (-3)^2 \cdot 4^2}$ f) $\dfrac{2^{-5} \cdot 8 \cdot 9 \cdot 3^{-2}}{2^{-4} \cdot 4^2 \cdot 6^{-1}}$

Place value

In our **decimal number system**, the **value** of each digit depends on its **place** in the number. Each place is 10 times the value of the next place to its right. Therefore, the decimal system is based upon **powers of ten**.

Millions	Hundred Thousands	Ten-Thousands	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	Ten-Thousandths
1,000,000	100,000	10,000	1,000	100	10	1	0.1	0.01	0.001	0.0001
10^6	10^5	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}

Examples:

7.4 → the 4 in 7.4 is in the *tenths* place. Its value is 4 *tenths*, or 0.4

74 → the 4 in 74 is in the *ones* place. Its value is 4 *ones*, or 4.

741 → the 4 in 741 is in the *tens* place. Its value is 4 *tens*, or 40.

7415 → the 4 in 7415 is in the hundreds place. Its value is 4 *hundreds*, or 400.

$$8{,}435 = 8{,}000 + 400 + 30 + 5 = 8 \cdot 10^3 + 4 \cdot 10^2 + 3 \cdot 10^1 + 5 \cdot 10^0$$

Exercise 29

Write the place and the value for each underlined.

	Place	Value
1.$\underline{3}$56		
2,65$\underline{7}$,009		
0.003$\underline{5}$6		
3$\underline{4}$7.15		
$\underline{5}$67.5		

Exercise 30

Make the largest number possible from the digits 6, 5, 8, 2, 5, 7.

Exercise 31

Find the number from these hints:

- it has more than 164 hundreds
- it has fewer than 17 thousands
- it contains the digits 6, 1, 9, 2, 5
- it has an even number of tens.

1.4.- ROOTS

Square roots

$\sqrt{25} = 5$ because $5^2 = 25$ $\qquad$ $\sqrt{\dfrac{25}{9}} = \dfrac{5}{3}$ because $\left(\dfrac{5}{3}\right)^2 = \dfrac{25}{9}$

Notice that there are two square roots of a positive number.

$$\sqrt{25} = 5 \text{ and } (-5) \text{ because } 5^2 = 25 \text{ and } (-5)^2 = 25$$

You can write: $\sqrt{25} = \pm 5$

The square root of a negative number doesn't exist, because a positive number to the power of two is a positive number and a negative number to the power of two is also a positive number.

$$\sqrt{-16} \neq 4 \qquad \sqrt{-16} \neq -4$$

Cube roots

$\sqrt[3]{8} = 2$ because $2^3 = 8$ $\qquad$ $\sqrt[3]{\dfrac{8}{1000}} = \dfrac{2}{10}$ because $\left(\dfrac{2}{10}\right)^3 = \dfrac{8}{1000}$

Nth roots

$\sqrt[5]{32} = 2$ because $2^5 = 32$ $\qquad$ $\sqrt[4]{10\,000} = 10$ because $10^4 = 10\,000$

In general, $\sqrt[n]{a} = b \Leftrightarrow b^n = a$

Exercise 32

Calculate the following roots:

a) $\sqrt[6]{64}$ b) $\sqrt[3]{-216}$ c) $\sqrt{14\,400}$

d) $\sqrt[6]{\frac{1}{64}}$ e) $\sqrt[3]{\frac{64}{216}}$ f) $\sqrt[3]{\frac{3375}{1000}}$

Exercise 33

Calculate these using a calculator, giving your answers to 2 decimal places as appropriate.

a) $\sqrt[3]{86}$ b) $\sqrt[3]{2.7}$ c) $\sqrt[3]{-70}$ d) $\sqrt[3]{0.015625}$