Unit 2: NUMBERS AND THEIR USES (II)

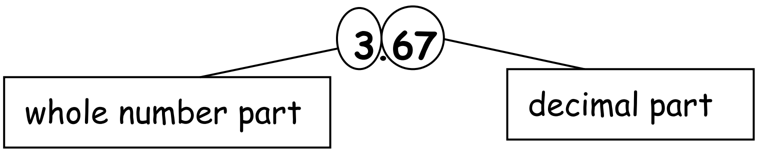
This unit will show you how to:

- Distinguish between recurring and terminating decimals
- Convert fractions to decimals and percentages and vice versa
- Calculate a percentage of a quantity
- Solve problems, involving percentage increase and decrease
- Recognise and manipulate surds
- Round numbers to any number of significant figures
- Round numbers to a given power of 10 or number decimals places
- Understand and use standard form in calculations with large and small numbers

Keywords	
Recurring decimal	Principal
Terminating decimal	Rate
Decimal equivalent	Rational number
Decrease	Irrational number
Increase	Surd
Interest	Round
Invest	Significant
Simple and compound interest	Standard form

2.1.- DECIMAL NUMBERS

A **decimal number** has two parts:

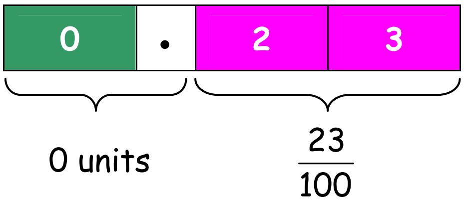


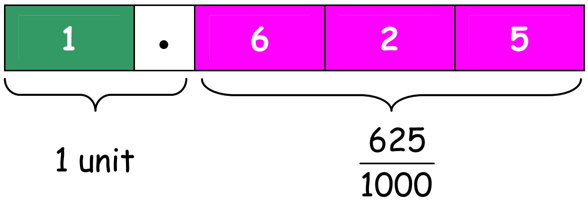
You say: three point six seven or three units and seventy-seven hundredths.

You can use a decimal place value diagram to see the value of each digit:

Tens	Ones		Tenths	Hundredths	Thousandths
1	**2**	**.**	**5**	**6**	**4**
1 ten is 10	2 units is 2		5 tenths is $\frac{5}{10}$	6 hundredths is $\frac{6}{100}$	4 thousandths is $\frac{4}{1000}$

You can write the decimal part of a number as a fraction:





$$0.23 = 0 + \frac{23}{100}$$

$$1.625 = 1 + \frac{625}{1000} = 1 + \frac{5}{8} = 1\frac{5}{8}$$

Example: Convert 0.225 and 4.32 to fractions in their simplest form.

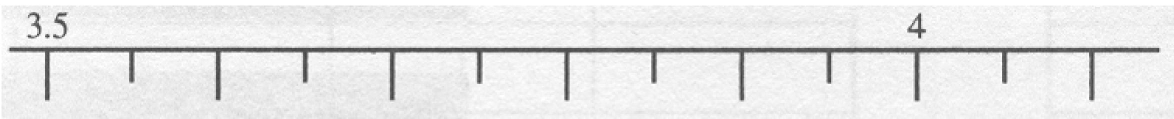
$$0.225 = \frac{225}{1000} = \frac{9}{40} \qquad 4.32 = 4 + \frac{32}{100} = 4 + \frac{8}{25} = 4\frac{8}{25}$$

Remember: you simplify the fraction by dividing top and bottom. Don't divide the whole number part.

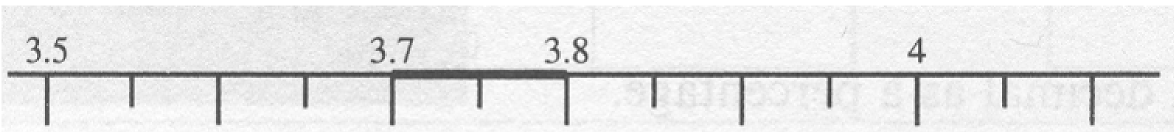
Decimals on a number line

To place a decimal on a number line you must decide which numbers it lies between.

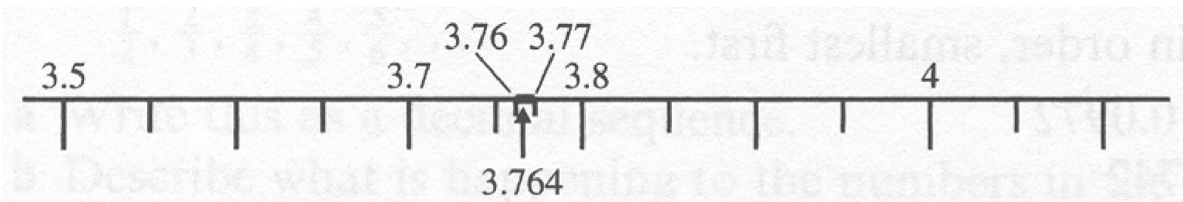
Example: Place 3.764 on this number line.



3.764 lies between 3.7 and 3.8 so find these on the line



3.764 lies between 3.76 and 3.77 so you must estimate.



Exercise 1

On copies of the number line above show the positions of:

a 3.74 **b** 3.98 **c** 3.625

Exercise 2

On a copy of the number line above show the position of: 4.052

Exercise 3

Draw a number line 10 cm long. Label one end 0.6 and the other 0.7. Show ten divisions between 0.6 and 0.7. On your line indicate the approximate position of:

a 0.665 **b** 0.6164 **c** 0.698

Types of decimal numbers

There are three different types of decimal number: exact, recurring and irrational numbers.

An **exact** or **terminating** decimal is one which does not go on forever, so you can write down all its digits. For example: 0.125.

A **recurring decimal** is a decimal number which does go on forever, but where some of the digits are repeated over and over again. For example: 0.12525252525… is a recurring decimal, where '25' is repeated forever.

Sometimes recurring decimals are written with a bar over the digits which are repeated, or with dots over the first and last digits that are repeated. For example: $3.2014014014... = 3.2\overline{014} = 3.2\dot{0}1\dot{4}$

Irrational numbers are those which go on forever and don't have digits which repeat. For example: $\sqrt{2} = 1.4142135...$, $\pi = 3.14159265...$

2.2.- FRACTIONS, DECIMALS AND PERCENTAGES

Converting fractions to decimals and percentages

- To convert a fraction to a decimal divide the numerator by the denominator.

Example: Write these fractions as decimals: a) $\frac{5}{8}$ b) $\frac{5}{9}$ c) $\frac{1}{12}$

a) $\frac{5}{8} = 0.625$ (terminating decimal)

b) $\frac{5}{9} = 0.555... = 0.\dot{5}$ (recurring decimal)

c) $\frac{1}{12} = 0.0833333... = 0.08\dot{3}$ (recurring decimal)

To decide if a fraction will be a terminating or a recurring decimal, look at the denominator.

- If the only factors of the denominator are 2 and/or 5 or combinations of 2 and 5 then the fraction will be a terminating decimal.
- It the denominator has any factors other than 2 and/or 5 then the fraction will be a recurring decimal.

Example: Say whether these fractions are terminating or recurring decimals.

a) $\frac{9}{20}$ b) $\frac{4}{30}$ c) $\frac{1}{12}$

a) denominator $= 20 = 2^2 \cdot 5 \rightarrow \frac{9}{20}$ is a terminating decimal

b) $\frac{4}{30} = \frac{2}{15}$ denominator $= 15 = 3 \cdot 5 \rightarrow \frac{2}{15}$ is a recurring decimal

c) denominator $= 12 = 2^2 \cdot 3 \rightarrow \frac{1}{12}$ is a recurring decimal

- <u>To convert a fraction to a percentage</u>:
  - write it as a decimal
  - multiply the decimal by 100%.

Example: Write as percentages

a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{2}{5}$ d) $\frac{7}{9}$

a) $\frac{1}{2} = 0.5$; $0.5 \times 100\% = 50\%$

b) $\frac{3}{4} = 0.75$; $0.75 \times 100\% = 75\%$

c) $\frac{2}{5} = 0.4$; $0.4 \times 100\% = 40\%$

d) $\frac{7}{9} = 0.\dot{7}$; $0.\dot{7} \times 100\% = 77.\dot{7}\% \rightarrow 77.8\%$ to 1 dp

Converting decimals and percentages to fractions

- <u>To convert a terminating decimal to a fraction</u>:
  1. Write the decimal as a fraction with denominator 10, 100, 1000, …, according to the number of decimal places.

For example: $0.45 = \frac{45}{100}$

2. Simplify the fraction.

$$0.45 = \frac{45}{100} = \frac{9}{20}$$

- <u>To convert a recurring decimal to a fraction</u>:

Look at these examples:

5.454545…

1. Let $x = 5.454545...$ (A)
2. Multiply by 100 (because there are two recurring figures)
   (If there were three recurring figures, you would multiply by 1000)
   $$100x = 545.454545... \quad (B)$$
3. Subtract B - A
   $$\begin{aligned} 100x &= 545.454545... \\ x &= 5.454545... \\ \hline 99x &= 540 \end{aligned}$$
4. Divide by 99
   $$x = \frac{540}{99} = \frac{60}{11}$$

2.5636363…

1. Let $x = 2.5636363...$
2. Multiply by 10 (because there are one figure between the whole part and the recurring figures)
   (If there were two figures between the whole part and the recurring figures, you would multiply by 100)
   $$10x = 25.636363... \quad (A)$$
3. Multiply (A) by 100 (because there are two recurring figures)
   $$1000x = 2563.636363... \quad (B)$$
4. Subtract B – A
   $$\begin{aligned} 1000x &= 2563.636363... \\ 10x &= 25.636363... \\ \hline 990x &= 2538 \end{aligned}$$

5. Divide by 990

$$x = \frac{2538}{990} = \frac{141}{55}$$

- <u>To convert a percentage to a fraction</u>, divide the percentage by 100.

For example: 25% $\rightarrow \frac{25}{100} = \frac{1}{4}$

Exercise 4

State whether each of these fractions will give a recurring decimal or a terminating decimal. Explain your answers.

a) $\frac{1}{25}$ b) $\frac{3}{20}$ c) $\frac{4}{11}$ d) $\frac{7}{15}$

Exercise 5

Shula says, 'I used my calculator to change $\frac{1}{13}$ to a decimal, and I got the answer 0.07692308. There is no repeating pattern, so the decimal does not recur.' Explain why Shula is wrong.

Exercise 6

Convert these decimals to fractions. Give your answers in their simplest form.

a) 0.32 b) 4.5 c) 5.3333... d) 8.35555...

Exercise 7

Copy and complete these tables (simplify any fractions).

Percentage	Fraction	Decimal
12%		
		0.36
	$\frac{3}{10}$	
25%		
		0.4
	1	

Percentage	Fraction	Decimal
	$\frac{4}{25}$	
75%		
		0.8
	$\frac{3}{5}$	
85%		
		0.15

Exercise 8

In each of these questions express the answer first as a fraction, then convert the fraction to a percentage using an appropriate method.

a) In a survey of 80 people, 55 said they would prefer full-time education to be compulsory until the age of 18. What percentage of the 80 people preferred full-time education to be compulsory until the age of 18?

b) In a football squad of 24 players, 5 of the players are goalkeepers. What percentage of the football squad are not goalkeepers?

Exercise 9

Leon scores 68% in his French exam and gets $\frac{37}{54}$ in his German exam. In which subject did he do the best? Explain your answer.

2.3.- PERCENTAGE CALCULATIONS

Finding a percentage of a quantity

You often need to calculate a percentage of a quantity.

- Use mental methods to find simple percentages
  Example: 50% of 400.
  50% of a quantity is the same as one half of that quantity, so 50% of 400 are 200.

- Change the percentage to its equivalent fraction and multiply by the amount.
  Example: 9% of 24 m.
  9% of 24 m $= \frac{9}{100} \times 24 = \frac{9 \times 24}{100} = \frac{216}{100} = 2.16$

- Change the percentage to its equivalent decimal and multiply by the amount.
  Example: 37% of £58.
  37% of £58 $= \frac{37}{100} \times 58 = 0.37 \times 58 = £21.46$
  (0.37 is the **decimal equivalent** of 37%)

Exercise 10

Calculate

a) 24% of 300 b) 3% of 450 c) 112% of 560 d) 4.5% of 100 000

Exercise 11

Paul downloads a file from the internet. The file is 16 Mb. After 2 minutes he has downloaded 70% of the file. How much of the file has Paul downloaded?

Exercise 12

A train journey is 395 km long. Lola is travelling on a train that has completed 23% of the journey. How many kilometres has Lola's train travelled?

Percentage increase and decrease

Percentages are used in real life to show how much an amount has increased or decreased.

- To calculate a **percentage increase**, work out the increase and add it to the original amount.
- To calculate a **percentage decrease**, work out the decrease and subtract it from the amount.

Examples:

a) Alan is paid £940 a month. His employer increases his wage by 3%. Calculate the new wage Alan is paid each month.

Increase in wage = 3% of £940 $= 0.03 \times £940 = £28.20$

Alan's new wage = $£940 + £28.20 = £968.20$

b) A new car costs £19 490. After one year the car depreciates in value by 8.7%. What is the new value of the car?

Depreciation = 8.7% of £19 490 $= 0.087 \times £19\,490 = £1695.63$

New value of car = $£19\,490 - £1695.63 = £17\,794.37$

<u>You can also calculate a percentage increase or decrease in a single calculation</u>.

Examples:

a) In a sale all prices are reduced by 16%. A pair of trousers normally costs £82. What is the sale price of the pair of trousers?

Sale price = $(100-16)\%$ of the original price = 84% of £82 =

$= 0.84 \times £82 = £68.88$

To find a result of a 16% decrease, multiply the original amount by the decimal equivalent of $(100-16)\%$, that is 0.84.

0.84 is the **decimal equivalent** of a 16% decrease.

b) Last year, Leanne's Council Tax bill was £968. This year the local council has raised the bill by 16%. How much is Leanne's new bill?

New bill = $(100+16)\%$ of the original bill = 116% of £968 =

$$= 1.16 \times £968 = £1122.88$$

To find a result of a 16% increase, multiply the original amount by the decimal equivalent of $(100+16)\%$, that is 1.16.

1.16 is the **decimal equivalent** of a 16% increase.

Exercise 13

Write the decimal number you must multiply by to find these percentages increases.

a) 20% b) 30% c) 45% d) 85% e) 6.5%

Exercise 14

Write the decimal number you must multiply by to find these percentages decreases.

a) 40% b) 60% c) 35% d) 72% e) 18.5%

Exercise 15

The price of a coat was £185. The price is reduced by 10% in a sale. What is the sale price of the coat?

Exercise 16

A house is bought for £195 000. During the next year, the house increases in price by 28.3%. What is the new value of the house?

Exercise 17

A lorry carries a load of sand that weighs 2.8 tonnes. The lorry loses 2.3% of its load as it travels. What mass of sand does the lorry now carry?

Exercise 18

Helen has a contract for her phone home. She pays £38.29 for calls and a quarterly charge of £19.60. VAT (Value Added Tax) has to be added at 17.5%. Calculate the cost of Helen's bill + VAT.

(VAT is a tax which is added to bills for services and purchases)

When people buy and sell things they try to sell for more than they paid.

The difference between the selling price and cost price is called the **profit**. A profit is normally written as a percentage of the cost price.

$$\% \text{ profit} = \frac{\text{profit}}{\text{cost price}}$$

Example:
Carina buys and sells protractors. She buys each protractor for 12p and sells them for 15p. What is her percentage profit?

Calculate the profit: $\text{Profit} = 15\text{p} - 12\text{p} = 3\text{p}$

Write the profit as a proportion of the cost price: $\frac{3\text{p}}{12\text{p}}$

Express this fraction as a percentage: $\frac{3}{12} = 0.25 = 25\%$

Reverse percentages

In a reverse percentage problem, you are given an amount after a percentage change, and you have to find the **original** amount.

Examples:
a) In a sale, a pair of shoes cost £38.25 after a 15% decrease. Find the original price of the shoes.

Sale price is $(100 - 15) = 85\%$ of the original price

$85\% = 0.85$

Original price $\times 0.85 =$ sale price

$\text{Original price} = \frac{£38.25}{0.85} = £45$

b) A table costs £88, including 17.5% VAT. Find the cost (to the nearest pound) before VAT was added.

£88 is 117.5% of cost before VAT $117.5\% = 1.175$

Cost before VAT $\times 1.175 =$ £88

$\text{Cost before VAT} = \frac{£88}{1.175} \simeq £75$ to the nearest pound

To sum up:

$$\boxed{\text{Original amount}} \times \boxed{\text{Decimal equivalent of increase or decrease}} = \boxed{\text{Final amount}}$$

$$\text{Original amount} = \frac{\text{Final amount}}{\text{Decimal equivalent of increase or decrease}}$$

Exercise 19

Calculate the original price of a hat that costs £46.50 after a 7% price cut.

Exercise 20

Bertha's pension was increased by 5.15% to £82.5. What was her pension before this increase?

Exercise 21

To decrease an amount by 8%, multiply it by 0.92. For a further decrease of 8%, multiply by 0.92 again, and so on. Use this idea to calculate

a) the final price of an item with an original price of £380, which is given two successive price cuts of 8%.

b) the final price of an item with an original price of £2400, which is given three successive price cuts of 10%.

Exercise 22

A company buys a van at a cost of £15 000. Each year the van depreciates in value by 17%. Work out the value of the van after 2 years.

(Some items reduce in value over time. This is called **depreciation**.)

Simple and compound interest

You earn **interest** when you **invest** in a savings account at a bank. However, you pay interest if you **borrow** money for a mortgage.

The original sum you invest is called the **principal**.

Interest is either

- **Simple interest** – it is not added to the **principal**, or
- **Compound interest** – added to the principal and will itself earn interest.

To calculate simple interest, use the interest **rate** to work out the amount earned.

If simple interest is paid for several years, the amount paid each time stays the same, because the interest is paid elsewhere and the principal stays the same.

To calculate compound interest, work out the interest in the same way, but add the interest earned to the principal.

If compound interest is paid for several years, the amount of interest earned each year increases, because the principal increases.

Examples:

a) Calculate the interest when £1000 is invested for 4 years at a 5% simple interest (SI).

SI for 1 year = 5% of $£1000 = 0.05 \times £1000 = £50$
SI for 4 years = $£50 \times 4 = £200$
Total SI = £200
Principal + SI = £1000 + £200 = £1200

b) £2000 is invested at 6.5% compound interest. Find the principal after 15 years.

To increase by 6.5% multiply by decimal equivalent of 6.5% = 1.065

Principal at end of 1 year $= £2000 \times 1.065 = £2130$
Principal at end of 2nd year $= (£2000 \times 1.065) \times 1.065 =$
$= £2000 \times 1.065^2 = £2268.45$

After 15 years principal $= £2000 \times 1.065^{15} = £5143.68$

Exercise 23

Find the total interest earned when these amounts of money are invested at these annual rates of simple interest.

a) £1400 at 7.5% for 3 years b) £650 at 3.5% for 10 years

Exercise 24

Fin the decimal number you should multiply the principal by to find the final amount after earning compound interest at these rates.

a) 5% per year for 3 years b) 6.5% per year for 5 years

Exercise 25

Use your answers of exercise 24 to work out the final amount when

a) £5000 is invested for 3 years at 5% compound interest.
b) £800 is invested for 5 years at 6.5% compound interest.

2.4.- RATIONAL AND IRRATIONAL NUMBERS

Rational numbers

A **rational number** is any number that can be expressed as the quotient $\frac{a}{b}$ of two integers, with the denominator b not equal to zero.

For example: $\frac{3}{5}$, $\frac{-10}{3}$

Since b may be equal to 1, every integer corresponds to a rational number.

For example: $-3 = \frac{-3}{1}$

The set of all rational numbers is usually denoted $\mathbb{Q}$ (for *quotient*).

$$\text{Rational numbers}\begin{cases}\text{Integers}\\ \text{Fractions}\begin{cases}\text{Terminating decimals}\\ \text{Recurring decimals}\end{cases}\end{cases}$$

Irrational numbers

An **irrational number** is a number that cannot be written as a simple fraction. Equivalently, irrational numbers cannot be represented as terminating or repeating decimals (the decimal part goes on forever without repeating).

Example: π (pi) is an irrational number. The value of π is

3.1415926535897932384626433832795 (and more...)

There is no pattern to the decimals, and you cannot write down a simple fraction that equals π.

It is called **irrational** because it cannot be written as a **ratio** (or fraction), not because it is crazy!

Is $\sqrt{2}$ an irrational number? My calculator says the square root of two is

1.4142135623730950488016887242097,

but this is not the full story! It actually goes on and on, with no pattern to the numbers. You cannot write down a simple fraction that equals $\sqrt{2}$.

Many square roots, cube roots, etc. are also irrational numbers.

Exercise 26

Put the following numbers in the right place.

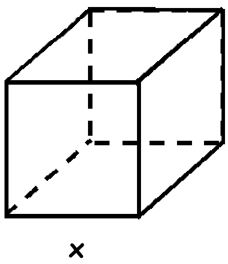
$$107\,;\;\; 3.95\,;\;\; 3.\dot{9}\dot{5}\,;\;\; -7\,;\;\; \sqrt{20}\,;\;\; \frac{36}{4}\,;\;\; \sqrt{\frac{4}{9}}\,;\;\; -\sqrt{36}\,;\;\; \frac{7}{3}\,;\;\; \pi-3$$

Natural numbers	
Integer numbers	
Fractions	
Rational numbers	
Irrational numbers	

2.5.- SURDS

When irrational numbers are written in a form using nth roots, they are called **surds** and they give the value exactly. Surds are exact answers, but their decimal equivalents are not.

Example: A cube has a volume of 10 cm^3. Find the length of one of its sides.



Let the length of a side be x cm

Volume = 10 cm^3

$x^3 = 10 \quad \Rightarrow \quad x = \sqrt[3]{10}$ cm

Rules for manipulating surds

There are rules you can use to manipulate and simplify surds. Here is a summary of those you need to know. When you use them, look for factors that are square numbers!

- $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$
- $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
- $m\sqrt{a} \cdot n\sqrt{b} = mn\sqrt{ab}$
- $\frac{m\sqrt{a}}{n\sqrt{b}} = \frac{m}{n}\sqrt{\frac{a}{b}}$
- $m\sqrt{a} + n\sqrt{a} = (m+n)\sqrt{a}$

Example 1: Work out each of the following expressions, leaving your answers in surd form.

a) $\sqrt{3} \cdot \sqrt{5} = \sqrt{15}$ b) $\frac{\sqrt{63}}{\sqrt{21}} = \sqrt{\frac{63}{21}} = \sqrt{3}$

c) $2\sqrt{11} \cdot 5\sqrt{2} = 10\sqrt{22}$ d) $\frac{12\sqrt{30}}{3\sqrt{6}} = \frac{12}{3}\sqrt{\frac{30}{6}} = 4\sqrt{5}$

Example 2: Simplify each of the following expressions, leaving your answers in surd form.

a) $\sqrt{72} = \sqrt{36 \cdot 2} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$

b) $\sqrt{\frac{96}{50}} = \frac{\sqrt{96}}{\sqrt{50}} = \frac{\sqrt{16 \cdot 6}}{\sqrt{25 \cdot 2}} = \frac{4\sqrt{6}}{5\sqrt{2}} = \frac{4}{5}\sqrt{\frac{6}{2}} = \frac{4}{5}\sqrt{3}$

c) $\sqrt{147} - \sqrt{48} = \sqrt{49 \cdot 3} - \sqrt{16 \cdot 3} = 7\sqrt{3} - 4\sqrt{3} = (7-4)\sqrt{3} = 3\sqrt{3}$

d) $\sqrt{252} - \sqrt{28} = \sqrt{36 \cdot 7} - \sqrt{4 \cdot 7} = 6\sqrt{7} - 2\sqrt{7} = 4$

Exercise 27

Work out each of the following expressions, leaving your answers in surd form.

a) $\sqrt{5} \cdot \sqrt{8}$ b) $\frac{\sqrt{30}}{\sqrt{5}}$ c) $3\sqrt{5} \cdot 2\sqrt{3}$ d) $\frac{6\sqrt{10}}{3\sqrt{2}}$

Exercise 28

Simplify each of the following expressions, leaving your answers in surd form.

a) $\sqrt{\frac{32}{50}}$ b) $\sqrt{\frac{24}{64}}$ c) $\sqrt{\frac{27}{63}}$ d) $\sqrt{\frac{48}{98}}$

e) $\sqrt{8} + \sqrt{8}$ f) $\sqrt{20} + \sqrt{45}$ g) $\sqrt{75} - \sqrt{48}$ h) $\sqrt{150} - \sqrt{96}$

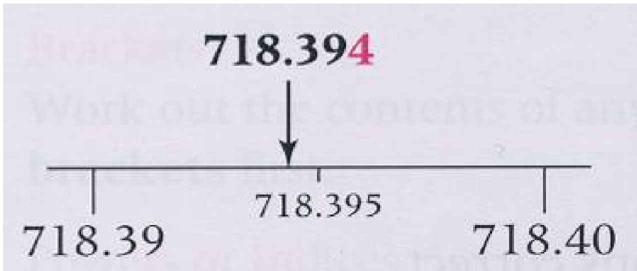
2.6.- APROXIMATION AND ROUNDING

Rounding a number is another way of writing a number approximately. We often don't need to write all the figures in a number, as an approximate one will do.

For a population of 27 653 the number is large and will change daily. It is better to round up and say 28 000.

When **rounding** numbers to a given degree of accuracy, look at the next digit. If it is **5** or more then round **up**, otherwise round down.

Example: To round 718.394 to 2 **decimal places**, look at the **thousandths** digit.



The **thousandths** digit is **4**, so round **down** to 718.39.

$718.394 \approx 718.39$ (to 2 decimal places)

Numbers can be rounded:

- to **decimal places** 4.16 = 4.2 to 1 decimal place
- to **the nearest unit, 10, 100, 1000, …**

32 559 = 33 000 to the nearest thousand

The first **non-zero** digit in a number is called the **1st significant figure** –it has the highest value in the number.

When rounding to **significant figures**, count from the first non-zero digit.

Examples:

$54.76 \approx 55$ (to 2 significant figures)

$0.00405 \approx 0.0041$ (to 2 significant figures)

$6.339 \approx 6.34$ (to 3 significant figures)

$0.000\,000\,338\,754 \approx 0.000\,000\,339$ (to 3 significant figures)

You can **estimate** the answer to a calculation by rounding the numbers.

Example: Estimate the answer to $\frac{6.23 \times 9.89}{18.7}$.

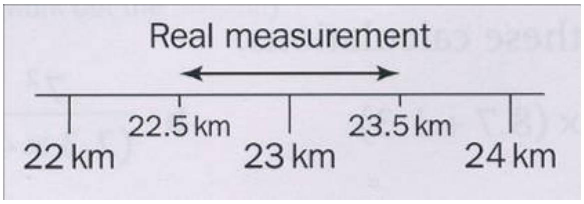
You can round each of the numbers to 1 significant figure.

$$\frac{6.23 \times 9.89}{18.7} \approx \frac{6 \times 10}{20} = 3$$

dp and **sf** are abbreviations for 'decimal places' and 'significant figures'.

When a measurement is written, it is always written to a given degree of accuracy. The real measurement can be anywhere within $\pm$ half a unit.

Example: A man walks 23 km (to the nearest km). Write the maximum and minimum distance he could have walked.



Because the real measurement has been rounded, it can lie anywhere between 22.5 km (minimum) and 23.5 (maximum).

Exercise 29

Round these decimal numbers to the nearest whole number.

a) 5.8　　b) 21.67　　c) 39.175

Exercise 30

Round these numbers to the nearest 1000.

a) 2239　　b) 12 563　　c) 155 669

Exercise 31

Round these decimal numbers to two decimal places (nearest hundredth).

a) 0.317 b) 15.304 c) 16.445

Exercise 32

Use a calculator to work these out. Write your answers correct to two significant figures.

a) $8 \div 13$ b) 7.8×71 c) 2093×3493

Exercise 33

Write a suitable estimate for each of these calculations. In each case, clearly show how you estimated your answer.

a) 4.98×6.12 b) $17.89 + 21.91$ c) $\dfrac{5.799 \times 3.1}{8.86}$

Exercise 34

For each of these measurements (given to a specified degree of accuracy), write the minimum and maximum value it could be.

a) 67.0 cm (nearest whole number)
b) 34.7 litres (1 decimal place)
c) 8.36 kg (2 decimal places)
d) 0.387 mm (3 decimal places)

2.7.- STANDARD INDEX FORM FOR LARGE NUMBERS

You can use standard form to represent large numbers.

In **standard form**, a number is written as $A \times 10^n$.

- A is a number between 1 and 10 (but not including 10). Using algebra, $1 \le A < 10$.
- The value of n is an integer.

For example, $43\,000\,000 = 4.3 \times 10^7$

Example: The Andromeda Galaxy has a radius of about 1 040 700 000 000 000 000 km. Write this in standard form.

$$1\,040\,700\,000\,000\,000\,000 = 1.0407 \times 10^{18}$$

You can calculate with numbers in standard form.

- Multiplication works like this:
$$\left(3\times10^5\right)\times\left(4\times10^3\right)=\left(3\times4\right)\times10^{5+3}=12\times10^8=1.2\times10^9$$
- Division works like this:
$$\left(1.4\times10^8\right)\div\left(7\times10^5\right)=\left(1.4\div7\right)\times10^{8-5}=0.2\times10^3=2\times10^2$$

You can work with numbers in standard form on a scientific calculator. The button for entering the power of 10 is often marked EXP or EE; so for 4.5×10^3, you might enter

4	•	5	EXP	3

Exercise 35

These numbers are in standard form. Write each of them as an 'ordinary' number.

a) 3.8×10^6 b) 7×10^{11} c) 9.73×10^8

Exercise 36

Use a scientific calculator to evaluate these. Give your answers in standard form, to 3 significant figures.

a) $\left(4.95\times10^3\right)\times\left(8.11\times10^7\right)$ b) $\left(3.7\times10^{11}\right)\div\left(1.8\times10^3\right)$

Exercise 37

Write these numbers in standard form.

a) Distance Earth-Sun: 150 000 000 b) Speed of light: 300 000 000 m/s

Exercise 38

A light-year is the distance that light travels in one year. Calculate that distance.

Exercise 39

Complete the table to show the time taken for light from the Sun to reach the various planets. (Hint: divide distance by speed)

Planet	Mean distance from Sun (m)	Light travel time
Earth	1.50×10^{11}	
Mars	2.28×10^{11}	
Jupiter	7.78×10^{11}	

2.8.- STANDARD FORM FOR SMALL NUMBERS

It is often useful to write small numbers, such as 0.000415, in standard form.

- Negative powers of 10, such as 10^{-4}, represent small numbers.

$$10^{-4} = \frac{1}{10^4} = \frac{1}{10\ 000} = 0.001$$

- You can write any small number in standard form.

$$0.00312 = 3.12 \times 0.001 = 3.12 \times 10^{-3}$$

Example: the diameter of an atom of gold is about

$$0.000\ 000\ 000\ 26\ \text{m} = 2.6 \times 10^{-10}\ \text{m}$$

- You can calculate with small numbers expressed in standard form.

$$\left(4.25 \times 10^{-7}\right) \div \left(3.75 \times 10^{-3}\right) = \left(4.25 \div 3.75\right) \times 10^{-7-(-3)} = 1.13 \times 10^{-4}$$

Exercise 40

These numbers are in standard form. Write each of them as an 'ordinary' number.

a) 5×10^{-4} b) 8.5×10^{-6} c) 4.13×10^{-8}

Exercise 41

Write this numbers in standard form.

a) 0.0047 b) 0.000 078 c) 0.000 000 000 067

Exercise 42

Use a scientific calculator to evaluate these. Give your answers in standard form, to 3 significant figures.

a) $\left(1.79 \times 10^{5}\right) \times \left(2.8 \times 10^{-12}\right)$ b) $\left(5.3 \times 10^{5}\right) \div \left(2.9 \times 10^{8}\right)$

Exercise 43

Use a scientific calculator to find the volume of a cube of side length 4.5×10^{-3} metres. Give your answer in m^3, to 3 sf, in standard form.

Exercise 44

A pack of 500 sheets of A4 paper weighs 2.65 kg. Find the mass in kg of a single sheet of paper, giving your answer in standard form.